Uniaxial and equi- biaxial tension tests of silicone elastomer

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ABSTRACT: At Technical University of Ilmenau a Peristaltically Actuated Device for Minimal Invasive Surgery (PADeMIS) is being developed. PADeMIS will be produced from silicone rubber and its design is being optimized by Finite Element Analysis (FEA). Thus, a constitutive law of silicone rubber is required. When an element of PADeMIS is hydraulically actuated the resulting deformation corresponds approximately to a biaxial tension. On the other hand, uniaxial tension tests are generally easier to perform than biaxial tension tests. Therefore, in this paper the validity of a Mooney- Rivlin law, adapted to one loading case and describing deformations during other loading cases is investigated. Furthermore, the results of fitting the parameters of the Mooney- Rivlin law to the data of equi- biaxial and uniaxial tension tests of silicone rubber simultaneously are presented.

1 INTRODUCTION

At Technical University of Ilmenau a Peristaltically Actuated Device for Minimal Invasive Surgery (PADeMIS) is being developed. The device will move actively like an earthworm and carries a hollow tube behind its back. The tube and the active part of the device are providing a channel to insert endoscopic tools towards the invasive location of surgery. The first application of PADeMIS is the minimal invasive spine surgery. It will enter the spinal canal at os sacrum and moves cranially between the vertebral bodies and the dura mater spinalis. Therefore, PADeMIS' design depends on the properties of the spinal canal. Thus, the outer diameter of PADeMIS must be alterable from 4 to 10 mm and an inner diameter of at least 2 mm has to be remained in order to insert the endoscopic instruments. Due to the general conditions PADeMIS will be produced from silicone rubber. It will consist of "worm"- segments, each made of at least two layers of silicone enclosing pads. The pads of serially arranged segments will be filled periodically with fluid, which is resulting in a peristaltic locomotion (see Figure 1). The design of a single segment is being optimized by Finite Element Analysis (FEA) (see Figure 2) for which a constitutive law as input

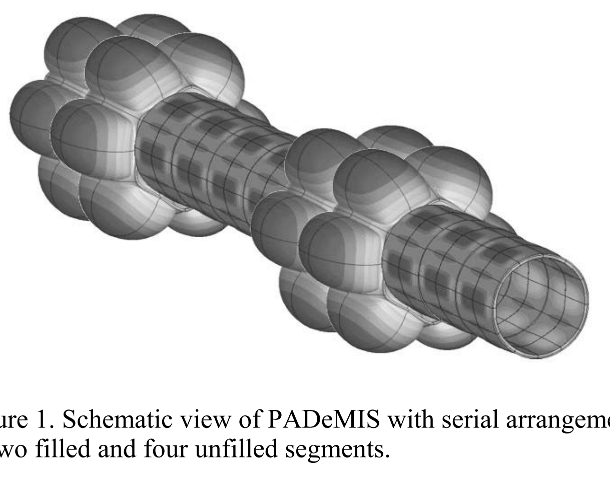

Figure 1. Schematic view of PADeMIS with serial arrangement of two filled and four unfilled segments.

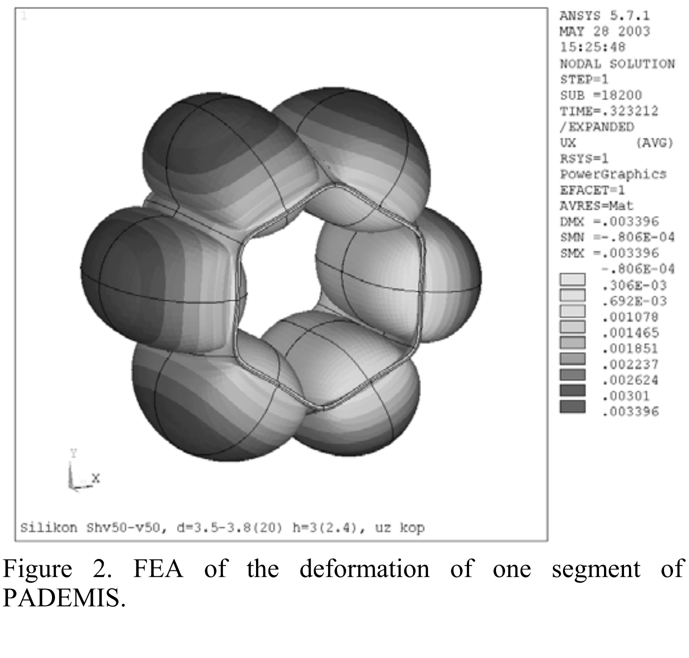

Figure 2. FEA of the deformation of one segment of PADEMIS.

property is necessary. The loading case of the deformed pads will be approximately biaxial. On the other hand, uniaxial tension tests are generally much easier to perform. However, in order to gain permission for medical use, many long-term experiments that are examining the mechanical fatigue and stress-softening (Mullins 1969) must be carried out.

The aim of this paper is to study whether a constitutive law (Mooney- Rivlin law) fitted to uniaxial tension tests can be used to describe biaxial tests. Or more general, whether fitting of a constitutive law to one specified loading case leads to a confidential prediction of deformations under other loading conditions. Furthermore a simultaneous fit of uniaxial and equi- biaxial experimental data is presented.

2 CONSTITUTIVE LAW - THEORY

To perform FEA simulations of the design of PADeMIS a constitutive law of the silicone rubber is needed as input quantity. A well known constitutive law for rubber like materials is the extended Mooney- Rivlin law, which is based on polynomials (Mooney 1940; Rivlin 1984):

$$W = \sum_{i+k=1}^{m} a_{ik} (I_1 - 3)^i (I_2 - 3)^k + \tfrac{1}{2}\kappa (I_3 - 1) \tag{1}$$

with W = strain energy density, I_1, I_2, I_3 = invariants of the deformation tensor, κ = bulk modulus, m = order of model and a_{ik} = the Mooney- Rivlin constants (q = number of parameters) describing the material. The invariants for incompressible material – a good assumption for silicone rubber – can be expressed as

$$\begin{aligned} I_1 &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2 \\ I_2 &= \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} + \frac{1}{\lambda_3^2} \\ I_3 &= \lambda_1^2 \lambda_2^2 \lambda_3^2 = \left(\frac{V}{V_0} \right)^2 = 1 \end{aligned} \tag{2}$$

with V = deformed volume, V_0 = undeformed volume and λ_i = principal stretches

$$\lambda_i = \frac{l_i}{l_{i,0}} \tag{3}$$

with l_i $l_{i,0}$ = deformed respectively undeformed length in direction i.

The principal true stresses σ_i can be calculated from the strain energy density by

$$\begin{aligned} \sigma_i &= -p + \lambda_i \frac{\partial W}{\partial \lambda_i} \\ &= -p + 2\lambda_i^2 \frac{\partial W}{\partial I_1} - \frac{2}{\lambda_i^2} \frac{\partial W}{\partial I_2} \end{aligned} \tag{4}$$

with the hydrostatic pressure p.

2.1 *Equi- biaxial tension*

For equi- biaxial stress in λ_1 and λ_2 and assuming $I_3 = 1$ the stretches in incompressible material can be expressed as

$$\begin{aligned} \lambda_2 &= \lambda_1 \\ \lambda_3 &= \frac{1}{\lambda_1^2} \end{aligned} \tag{5}$$

The surface perpendicular to direction λ_3 is stress-free. The hydrostatic pressure can be calculated by inserting Equation 4 in $\sigma_3 = 0$. Thus, from Equation 4 the stress $\sigma_{1,bi}$ can be calculated as

$$\sigma_{1,bi} = 2\left(\lambda_1^2 - \frac{1}{\lambda_1^4} \right) \left[\frac{\partial W}{\partial I_1} + \lambda_1^2 \frac{\partial W}{\partial I_2} \right] \tag{6}$$

Inserting the derivations of Equation 1 the stress can be expressed as

$$\begin{aligned} \sigma_{1,bi} = {} & 2\left(-\frac{1}{\lambda_1^4} + \lambda_1^2 \right) a_{10} \\ & + 2\left(-\frac{1}{\lambda_1^2} + \lambda_1^4 \right) a_{01} \\ & + 4\left(-\frac{1}{\lambda_1^8} + \frac{3}{\lambda_1^4} - \frac{1}{\lambda_1^2} - 3\lambda_1^2 + 2\lambda_1^4 \right) a_{20} \\ & + 6\left(-\frac{1}{\lambda_1^6} + \frac{1}{\lambda_1^4} + \frac{1}{\lambda_1^2} - \lambda_1^2 - \lambda_1^4 + \lambda_1^6 \right) a_{11} \\ & + 4\left(-\frac{2}{\lambda_1^4} + \frac{3}{\lambda_1^2} + \lambda_1^2 - 3\lambda_1^4 + \lambda_1^8 \right) a_{02} \\ & + \dots \end{aligned} \tag{7}$$

In the linear terms ($m = 1$ in Equation 1) the a_{01} term dominates for $\lambda_1 > 1$ the a_{10} term, in the second order terms the a_{02} term dominates and so on for higher orders of m.

For this reason, the determination of the parameters a_{ik} while fitting the experimental equi- biaxial data is expected to be considerably more accurate for the terms belonging to the invariant I_2 (a_{0k}) as for the terms belonging to the invariant I_1 (a_{i0}).

2.2 *Uniaxial tension*

For the uniaxial stress in λ_1 the stretches in incompressible material can be expressed as

$$\lambda_2 = \lambda_3 = \sqrt{1/\lambda_1} \tag{8}$$

The free surfaces perpendicular to directions λ_2 and λ_3 are stress- free. The hydrostatic pressure can be calculated by inserting Equation 4 in $\sigma_3 = 0$. Thus, from Equation 4 the principal stress $\sigma_{1,uni}$ can be calculated.

$$\sigma_{1,uni} = 2\left(\lambda_1^2 - \frac{1}{\lambda_1}\right)\left[\frac{\partial W}{\partial I_1} + \frac{1}{\lambda_1}\frac{\partial W}{\partial I_2}\right] \tag{9}$$

Inserting the derivations of Equation 1 the stress can be expressed as

$$\begin{aligned}\sigma_{1,uni} &= 2\left(-\frac{1}{\lambda_1} + \lambda_1^2\right)a_{10} \\ &+ 2\left(-\frac{1}{\lambda_1^2} + \lambda_1\right)a_{01} \\ &+ 4\left(-\frac{2}{\lambda_1^2} + \frac{3}{\lambda_1} + \lambda_1 - 3\lambda_1^2 + \lambda_1^4\right)a_{20} \\ &+ 6\left(-\frac{1}{\lambda_1^3} + \frac{1}{\lambda_1^2} + \frac{1}{\lambda_1} - \lambda_1 - \lambda_1^2 + \lambda_1^3\right)a_{11} \\ &+ 4\left(-\frac{1}{\lambda_1^4} + \frac{3}{\lambda_1^2} - \frac{1}{\lambda_1} - 3\lambda_1 + 2\lambda_1^2\right)a_{02} \\ &+ \ldots\end{aligned} \tag{10}$$

For uniaxial tension the term a_{10} dominates for $\lambda_1 > 1$ the a_{01} term in the linear terms ($m = 1$ in Equation 1), in the second order terms the a_{20} term dominates and so on for higher orders of m.

For uniaxial tension, the determination of the parameters a_{ik} by means of fitting the experimental data should be substantially more accurate for the terms belonging to the invariant I_1 (parameters a_{i0}) than for the terms concerning the invariant I_2 (parameters a_{0k}).

2.3 *Comparison of uniaxial and equi- biaxial stresses*

Comparing Equation 7 with Equation 10 the amount of stress due to the terms concerning the invariant I_1 is nearly equal, but the amount of stress due the invariant I_2 and the mixed terms are quite different.

So constitutive laws evaluated through fitting uniaxial tension experiments are expected to be inaccurate in the description of other stress cases (equi- biaxial stress or even uniaxial compression, which is equivalent to equi- biaxial stress).

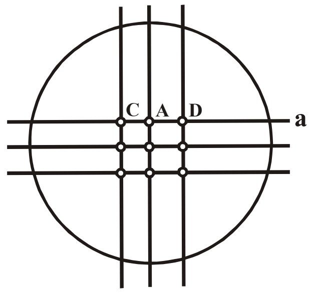

Figure 3. Top view of an undeformed equi- biaxial test sample. The positions of the crossing of the 6 lines will be measured, for example points C, A, D for line *a*.

3 EXPERIMENTAL SETUP

The segments of PADeMIS will be filled periodically with fluid to produce the locomotion. This results in nearly equi- biaxial stress in the silicone membrane (compare Figure 2). Therefore the constitutive law for the FEA optimization of the design should be evaluated by equi- biaxial tension tests. The disadvantages of the equi- biaxial tension tests are its intense time and manpower consumption. Hence, for longtime stability tests and investigations of stress softening (Mullins 1969) uniaxial tension tests are more comfortable.

3.1 *Silicone rubber test samples*

The samples are being produced from the liquid injection molding silicone elastomer MED-49xx from NUSIL® distributed by Polytec®. xx indicates the shore hardness of the silicone rubber adjusted with silica filler by the manufacturer. In this paper results from experiments performed with MED-4930 and MED-4950 are used. The two components are mixed in a 25% Hexane solution steadily, are casted in a mold of size 120 mm × 120 mm and are degassed for more than 30 hours. Then, the silicone rubber will be cured. The thickness of the sheet is measured with a layer thickness measurement Dualscope® made by Fischer®. The used layers had a thickness in the range from 500 - 900 µm and the standard deviation was less then 10%.

3.2 *Equi- biaxial tension test*

The equi- biaxial stress can be measured by inflating a thin silicon sheet as described by Rivlin and Saunders (Rivlin 1951). The silicon test sample will be marked with 6 lines as shown in Figure 3 and fixed to a aluminum plate by a aluminum ring of diameter 60 mm (see Figure 4). In the middle of the aluminum plate a pressure supply and a connection for a pressure sensor are included. Then the aluminum plate with the silicone sheet is mounted at a three axes positioning unit made by ISEL®.

On the vertical axes a tip is fixed (see Figure 4). Aligning this tip with the crossings of the marker lines the positions of the crossings can be measured.

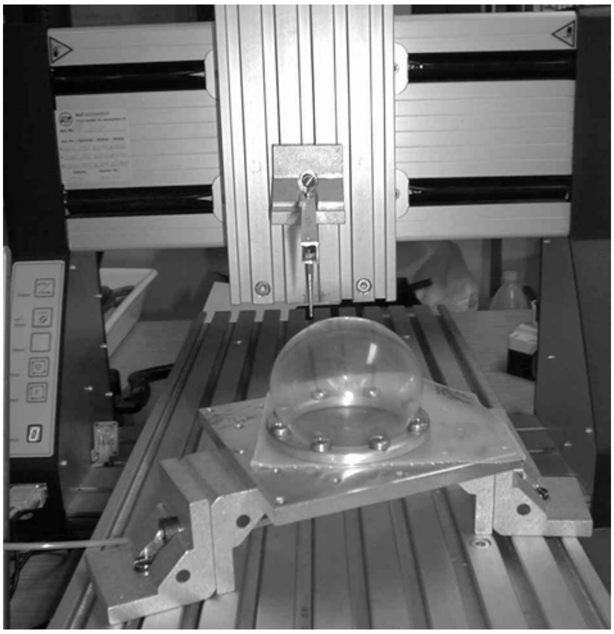

Figure 4. Equi- biaxial tension test stand

Inflating the sheet with pressure P causes a deformation. At the top of the bubble the deformation is assumed to be spherical with radius r. The pressure P produces a radial tension T:

$$P = \frac{2T}{r} \tag{11}$$

The radial tension is defined as

$$T = \int \sigma_{1,bi}\, dl \tag{12}$$

Assuming homogeneous stress in the sheet with deformed thickness l_3 Equation 12 can be simplified to

$$P = \frac{2\sigma_{1,bi}\, \mathrm{l}_3}{r} \tag{13}$$

The deformed length l_3 can be expressed by the undeformed length $l_{3,0}$ and the stretch λ_1:

$$\frac{l_3}{l_{3,0}} = \lambda_3 = \frac{1}{\lambda_1^2} \tag{14}$$

Thus, the stress in the sample can be expressed as

$$\sigma_{1,bi} = \sigma_{2,bi} = \frac{rP\lambda_1^2}{2l_{3,0}} \tag{15}$$

As shown by Rivlin and Saunders (Rivlin 1951) the radius r can be calculated from three points by the following geometrical considerations (nomenclature can be seen in Figure 5).

$$\Theta = \Phi + \Psi \tag{16}$$

and

$$\sin\frac{\Theta}{2} = \frac{\frac{1}{2}\overline{CD}}{r}$$
$$\sin\frac{\Phi}{2} = \frac{\frac{1}{2}\overline{CA}}{r} \tag{17}$$
$$\sin\frac{\Psi}{2} = \frac{\frac{1}{2}\overline{AD}}{r}$$

Inserting Equation 16 into Equation 17 and transforming it yields to

$$r = \frac{\sqrt{\left(x_1^2 + y_1^2\right)\left(x_2^2 + y_2^2\right)}}{2\left(x_1 y_2 + y_1 x_2\right)} \cdot \sqrt{\left(x_1 + x_2\right)^2 + \left(y_1 - y_2\right)^2} \tag{18}$$

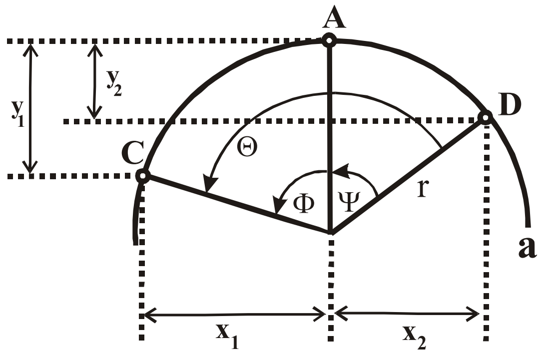

Figure 5. Side view of the line *a* indicated at Figure 3 of the deformed test sample. From the positions of the points A, C, D the stretch λ and the radius r will be calculated (see (Rivlin 1951)).

The stretch can then be expressed by

$$\lambda_1 = \frac{r\Theta}{l_{1,0}} \tag{19}$$

The angle Θ can be calculated by transforming Equation 17

$$\Theta = 2\arcsin\left(\frac{\sqrt{\left(x_1 + x_2\right)^2 + \left(y_1 - y_2\right)^2}}{2r}\right) \tag{20}$$

The stress $\sigma_{1,bi}$ for each sample will be calculated by Equation 15 for each line (see Figure 3). Then the mean of the stresses and stretches for the three parallel lines are calculated. The stress $\sigma_{2,bi}$ and the stretch λ_2 are calculated identically and are plotted separately. The difference between the stresses and stretches in both directions is an indicator for the inhomogeneity of the sample. The errors of the stretches and stresses are calculated by error propagation.

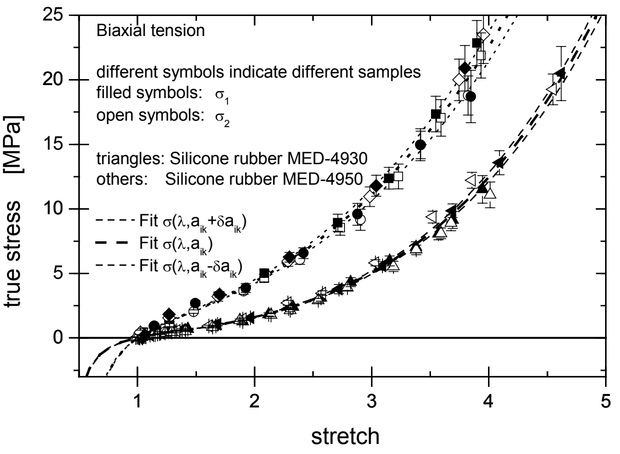

Figure 6. Experimental equi- biaxial tension test for silicone rubber MED-4930 and MED-4950.

Table 1. Mooney- Rivlin parameters of silicone rubber MED-4930 and MED-4950. Not listed parameters are equal zero. Fit explanation of χ^2 and ν see appendix.

Parameters	a_{10}*	δa_{10}*	a_{01}*	δa_{01}*	a_{20}*	δa_{20}*	χ^2	ν
MED-4950								
Biaxial fit	503	12	13	1			73	46
Uniaxial fit 1	497	6			0.9	0.2	718	217
Uniaxial fit 2	484	19	134	27	0.5	0.3	1003	216
Simultaneous bi- and uniaxial fit	493	5	10	1	0.9	0.2	800	264
MED-4930								
Biaxial fit	144	3	15	0.5			283	58
Uniaxial fit	245	6			2	0.1	87	432
Simultaneous bi- and uniaxial fit	192	2	1.3	0.4	3	0.05	544	492

*[kPa]

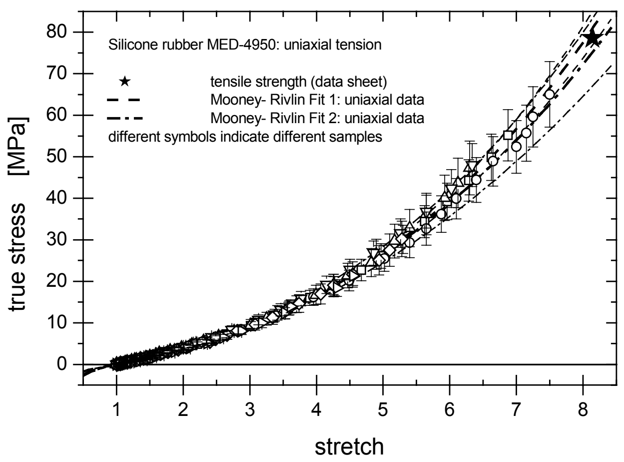

Figure 7. Experimental data and fits of the uniaxial tension tests for different samples of silicone rubber MED-4950.

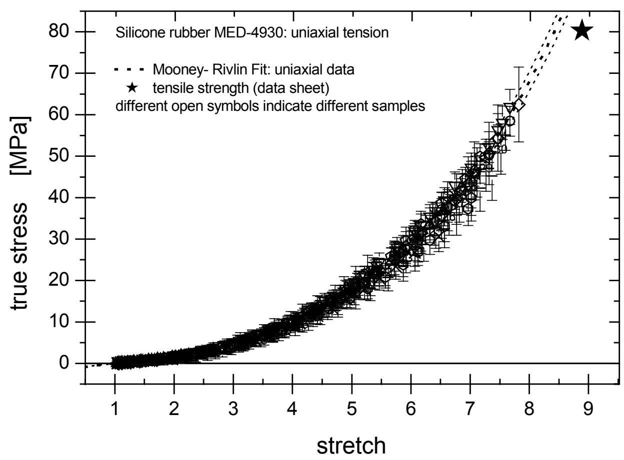

Figure 8. Experimental data and fits of the uniaxial tension tests for different samples of silicone rubber MED-4930.

In Figure 6 the measured stress- stretch dependency for equi- biaxial tension of silicone rubber MED-4930, MED-4950 and the fitted stress- stretch relation are shown. For details of the fitting process refer to the appendix.

As it can be seen, the stresses and stretches in direction 1 and 2 are identical within a 10% margin. Thus, the reproductively of measurements with different samples is very good. The experimental data can be well approximated by fitting Equation 7. The values of the fitted parameters can be seen in Table 1. There was no acceptable fitting result with parameters $a_{20} \neq 0$.

3.3 *Uniaxial tension test*

For the uniaxial tension test the silicone rubber sheet is cut in 80 mm × 5 mm long pieces. A 20 mm long region is marked around the center. The sample is fixed in clamps and attached to the test stand. Then, the sample is loaded with different weights and the resulting length is measured.

The principal stress $\sigma_{1,uni}$ in the sample can be calculated by

$$\sigma_{1,uni} = \frac{F}{A_{23}} = \frac{F}{A_{23,0}} \lambda_1 \qquad (21)$$

with F = force due to weight, $A_{23,0}$, A_{23} = undeformed respectively deformed area perpendicular to direction λ_1.

Estimation of the errors and fitting process are equivalent to the equi- biaxial experiments.

The experimental data of the uniaxial tension tests of silicone rubber MED-4950 and MED-4930 is shown in Figure 7 and Figure 8 respectively. For uniaxial tension of MED-4950 two fit curves are shown. For both materials the fitted stress- stretch characteristic closely matches the tensile strength specified in the data sheet of the manufacturer.

With regard to χ^2 and $\delta a_{01} / a_{01}$ and $\delta a_{20} / a_{20}$ fit 2 of material MED-4950 is much worse as fit 1. For both materials values of the parameters a_{01} could not be found. Thus, the assumption stated in the last paragraph chapter 2.2 – fitting of uniaxial tension tests results to uncertainties for the parameters a_{0k} – seems to be validated. The values of the fitted parameters can be seen Table 1.

3.4 *Optimization of the constitutive law to equi-biaxial and uniaxial tension tests*

The fitted constitutive laws are used to calculate a stress- stretch characteristic to predict the other loading case. The calculated stress- stretch relations for biaxial and uniaxial loading for silicone rubber MED-4930 and MED-4950 are shown in Figure 9 and Figure 10 respectively. In the MED-4950 biaxial loading case the errors between the stress- stretch characteristics calculated from the parameters of fit 2 from the uniaxial data and the experimental data are enormous. Also, a constitutive law fitted to the biaxial loading case cannot estimate the MED-4930 uniaxial data. From Table 1 it can be seen that a constitutive law for the silicone rubbers MED-4950 and MED-4930 needs (at least) the parameters a_{10}, a_{01} and a_{20}, but fits to one loading case give no satisfying results for all three parameters.

Thus, the LabVIEW™ Levenberg- Marquardt χ^2 minimization subroutine is modified to optimize the constitutive law to equi- biaxial and uniaxial data simultaneously:

$$\chi^2 = \chi^2_{uni} + \chi^2_{bi} \tag{22}$$

$\chi^2_{uni} = \chi^2$ of the uniaxial fitting procedure and $\chi^2_{bi} = \chi^2$ of the biaxial fitting procedure (calculated by Equation 23 shown in the Appendix).

The stress- stretch relations fitted to biaxial and uniaxial simultaneously are shown in Figure 9 and Figure 10 respectively, too. Thus, it has been shown that the Mooney- Rivlin approach of the constitutive law is able to describe the biaxial and uniaxial loading case simultaneously. The parameters a_{ik} of silicone rubber MED-4930 and MED-4950 are listed in Table 1. Via the simultaneous fit it is possible to find the three parameters a_{10}, a_{01}, a_{20} with small errors and acceptable χ^2. The validity of the resulting constitutive law for other loading cases can be estimated by calculating and transforming the stress corresponding to Equation 7 and Equation 10 and considering the orders of λ_1 for each parameter.

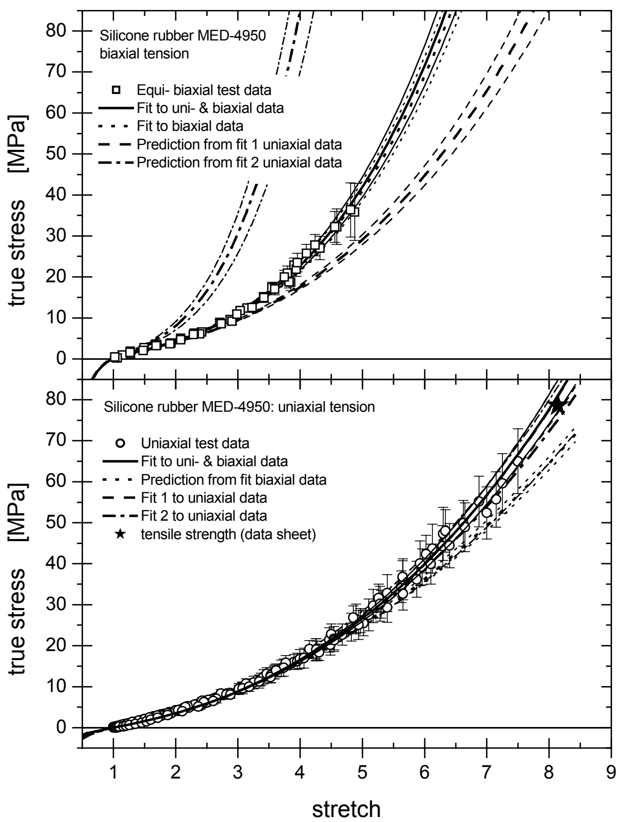

Figure 9. Fitted constitutive law for equi- biaxial and uniaxial tension tests of silicone rubber MED-4950.

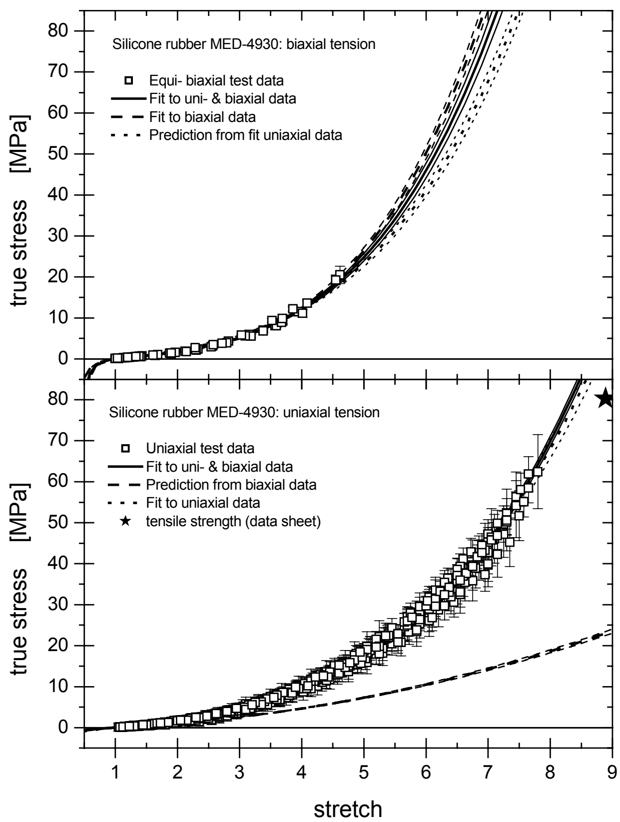

Figure 10. Fitted constitutive law for equi- biaxial and uniaxial tension tests of silicone rubber MED-4930.

4 CONCLUSIONS

On the one hand side, it is shown that fitting experimental data of one loading case can lead to wrong parameters in the Mooney- Rivlin law and therefore to wrong prediction for other loading cases. On the other hand, for silicone rubber a set of Mooney-Rivlin parameters describing uniaxial and equi-biaxial experiments simultaneously were found.

For FEA simulation the constitutive law, which is used as input, should be determined by a similar loading case to the one the simulated object will undergo. The other possibility is determining the constitutive law with several experiments of different loading cases.

In the future, models using orthogonal invariants (e.g. (Criscione 2000)) and models dealing with stress softening (e.g. (Ogden 1999)will be tested for their adaptability for describing more then one loading case simultaneously.

5 ACKNOWLEDGEMENT:

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6 APPENDIX

The experimental data are fitted with a LabVIEW™ program. A modified version of the LabVIEW™ Levenberg- Marquardt χ^2 minimization subroutine from National Instruments Corporation® is used (For programming Levenberg- Marquardt method see i.e. (Press 2002)).

The property χ^2 describing the distance between the fitted curve and the experimental value is defined as

$$\chi^2 = \sum^n \left(\frac{\sigma_{fit}(\lambda, a_{ik}) - \sigma_{meas}}{\delta\sigma_{meas}} \right)^2 \tag{23}$$

$\sigma_{fit}(\lambda,a_{ik})$ = calculated constitutive law (Equation 7 and Equation 10 respectively), σ_{meas} = stress from measured data (Equation 15 and Equation 21 respectively), $\delta\sigma_{meas}$ = experimental errors and n number of experimental data. In addition $\sigma_{fit}(\lambda,a_{ik} + \delta a_{ik})$ and $\sigma_{fit}(\lambda,a_{ik} - \delta a_{ik})$ are calculated to get an impression of the errors of the fitting process.

To calculate the errors δa_{ik} of the parameters a_{ik} a vector $\boldsymbol{c}$ with elements c_j has to be calculated.

$$c_j = \frac{\partial\sigma(\lambda, a_{ik})}{\partial a_{ik}} \tag{24}$$

The dimension of vector $\boldsymbol{c}$ responds to o, the number of parameters a_{ik}.

The convergence matrix $\boldsymbol{C}$ (dimension $q \times q$) is defined as

$$\mathbf{C} = \left(\sum^n \frac{\mathbf{c}^T \mathbf{c}}{\delta\sigma^2} \right)^{-1} \tag{25}$$

The error δa_{ik} of the j-th parameter is related to the diagonal elements of the Matrix $\boldsymbol{C}$ by

$$(\partial a_{ik})_j = \sqrt{C_{jj}} \tag{26}$$

The non- diagonal C_{jl} elements are the covariances between the j-th and l-th parameters.

Besides, the degree of freedom ν of the fit is specified as

$$\nu = n - q \tag{27}$$

A good fit should result in $\chi^2 \approx \nu$.

The fitting of the experimental data is performed by optimizing the parameters belonging to m = 1 (Equation 1) (The other parameters fixed to zero). Then the parameters belonging to m = 2 are included in the optimization subsequently. Parameters with errors larger then the value of the parameter are fixed to zero and the optimization was repeated. Parameters belonging to m > 2 never yield to better fit results.

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